B.Sc. Part–III (Semester–V) Examination

MATHEMATICS

(Mathematical Methods)

Paper—X

Time : Three Hours] [Maximum Marks : 60

Note :— (1) Question No. **1** is compulsory and attempt it once.

(2) Solve **ONE** question from each unit.

1. Choose the correct alternatives :—

(1) The integral $\int_{-1}^{1} x^m . p_n(x)\, dx = 0$ if _____.

(a) $m > n$ (b) $m < n$

(c) $m = n$ (d) $m \neq n$

(2) The eigen value of S-L problem are ______.

(a) Real (b) Complex

(c) Equal (d) None of these

(3) The value $J_{-\frac{1}{2}}(x) =$ ______.

(a) $\sqrt{\frac{2}{\pi x}} \sin x$ (b) $\sqrt{\frac{\pi x}{2}} \sin x$

(c) $\sqrt{\frac{2}{\pi x}} \cos x$ (d) $\sqrt{\frac{\pi x}{2}} \cos x$

(4) Fourier series for odd function is _____.

(a) $f(x) = \sum_{n=1}^{\infty} \sin n \frac{\pi x}{\ell}$ (b) $f(x) = \sum_{n=1}^{\infty} b_n \sin n \frac{\pi x}{\ell}$

(c) $f(x) = \sum_{n=1}^{\infty} b_n \sin n \pi x$ (d) None of these

(5) Inverse L.T. of $\frac{1}{S^n}$ = ______, when 'n' is positive integer.

(a) $\frac{1}{(n-1)!}$ (b) $\frac{t^n}{(n-1)!}$

(c) $\frac{t^{n-1}}{(n-1)!}$ (d) None of these

(6) The equation $\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$, where 'c' is constant is called ______.

(a) Wave equation (b) Heat equation

(c) Heat conduction (d) None of these

(7) The value of $F[e^{-|x|}]$ = _______.

(a) $\frac{1}{1+\lambda^2}$ (b) $\frac{2}{1-\lambda^2}$

(c) $\frac{2}{1+\lambda^2}$ (d) $\frac{1+\lambda^2}{2}$

(8) The finite Fourier sine and cosine transform and their inverses are ______ transformation.

(a) Linear (b) Bilinear

(c) Co-linear (d) None of these

(9) The value of $p_0(x) + 2p_2(x)$ is :

(a) $5x^2$ (b) $4x^2$

(c) $3x^2$ (d) $2x^2$

(10) If $L[f(t)] = F(s)$, then $L\left[\int_0^t F(u)\,du\right]$ is :

(a) $\frac{F(s)}{s}$ (b) $SF(s)$

(c) $\frac{F(s)}{a}$ (d) $aF(s)$ 10×1=10

UNIT—I

2. (a) Show that :

(i) $p_n(1) = 1$ and

(ii) $p_n(-x) = (-1)^n p_n(x)$

and hence deduce that

$p_n(-1) = (-1)^n$. 5

(b) Use Rodrigues formula to find $p_n(x)$, n = 0, 1, 2, 3, 4. 5

3. (p) Prove that :

$np_n = xp'_n - p'_{n-1}$, where $p'_n = \frac{d}{dx}p_n$. 5

(q) Prove that :

(i) $\int_{-1}^{1} p_n(x)\,dx = 0,\ n \neq 0$ and

(ii) $\int_{-1}^{1} p_o(x)\,dx = 2$. 3+2

UNIT—II

4. (a) Prove that :

$2n\ J_n(x) = x[J_{n-1}(x) + J_{n+1}(x)]$ 5

(b) Find all the eigenvalues and eigen functions of the Sturm-Liouville problem

$$y'' + \lambda^2 y = 0$$

where $y'(0) = y'(\ell) = 0, \quad 0 \le x \le \ell$. 5

5. (p) Prove that :

$J_{-n}(x) = (-1)^n J_n(x)$, if n is a positive integer. 5

(q) Prove that :

(i) $J_0'(x) = -J_1(x)$

(ii) $J_2 = J_0'' - x^{1} J_0^{1}$. 3+2

UNIT—III

6. (a) Obtain Fourier series for $\sqrt{1-\cos x}$ in $(0, 2\pi)$.

Hence deduce that

$$\sum_{n=1}^{\infty} \frac{1}{4n^2-1} = \frac{1}{2}.$$ 5

(b) Obtain the Fourier sine series for $f(x) = x^2$ in $0 < x < 2$. 5

7. (p) Find the Fourier expansion of

$$f(x) = \begin{cases} 1 + \dfrac{2x}{\pi} & , \quad -\pi < x < 0 \\ 1 - \dfrac{2x}{\pi} & , \quad 0 < x < \pi \end{cases}$$

Hence deduce that

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\ldots = \frac{\pi^2}{8}.$$ 5

(q) Find the Fourier Series to represent the function F(x) = |x| in the interval $-\pi < x < \pi$. Hence deduce that

$$\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+.......=\frac{\pi^2}{8} .$$ 5

UNIT—IV

8. (a) State and prove Convolution theorem. 5

(b) Solve the differential equation

$$\frac{d^2x}{dt^2}+9x=\cos 2t$$

by using Laplace transform, given by x(0) = 1; $x\left(\frac{\pi}{2}\right)=-1$. 5

9. (p) If $L^{-1}[F(s)] = F(t)$, then show that

$L^{-1}[F(s - a)] = e^{at} F(t)$. 4

(q) Using Laplace transform of derivative, prove that $L[e^{at}]=\frac{1}{S-a}$. 3

(r) Evaluate $\int_0^\infty e^{-3t}\, t\sin t\, dt$. 3

UNIT—V

10. (a) Find the finite Fourier sine and cosine transform of $f(x) = x^2$, $0 < x < \ell$. 4

(b) Find the Fourier transform of

$$f(x)=\begin{cases}1 & , \ |x| < 1\\ 0 & , \ |x| > 1\end{cases}$$

Hence deduce that $\int_{-\infty}^{\infty}\frac{\sin x}{x}dx=\frac{\pi}{2}$. 4

(c) Prove that :

$$\int_0^t f'(x)\sin\frac{n\pi x}{\ell}dx=-\frac{n\pi}{\ell}F_C(x).$$ 2

11. (p) Find the Fourier sine transform of u_x and u_{xx}; where u = u(x, t). 5

(q) Show that Fourier cosine transform of $F(x) = e^{-x^2}$ is $\frac{1}{\sqrt{2}}e^{-\frac{x^2}{4}}$. 5